PENMAN APPROACH TO EVAPORATION AND EVAPOTRANSPIRATION

1. Penman (1948, 1956) devised a method for estimating evaporation from free-water surfaces which combined energy-budget and mass-balance methods. By considering the evaporation from a sunken pan, he could ignore heat storage changes and conduction through the pan walls because these would be small.

2. Thus he could write:

   a. $Q_n = Q_h - Q_e$ (1)

   $$\left.\begin{array}{l} Q_n = \textit{net} \text{ solar radiation} \\ Q_h = \text{conduction from atmosphere} \\ Q_e = \text{energy used for evaporation} \end{array}\right\} \text{ in cal/cm}^2$$

   b. dividing both sides of the equation by ρL

   $$\frac{Q_n}{\rho L} = \frac{Q_h}{\rho L} + \frac{Q_e}{\rho L} \qquad (2)$$

   ρ = density of water (gm/cm^3)
   L = latent heat of vaporization (cal/gm)

   c. we can rewrite this as

   $H = K + E$ (3)

   $H = \dfrac{Q_n}{\rho L}$ = *net* amount of heat from sun, expressed as depth of water it could evaporate

   $K = \dfrac{Q_h}{\rho L}$ = *net* conduction of heat to/from atmosphere, expressed as depth of water it could evaporate

   E = evaporation (cm)

   d. the *Bowen Ratio* is the ratio of Q_h to Q_e and is given by

   $$R = \frac{Q_h}{Q_e} = \gamma \frac{(T_s - T_a)}{(e_s - e_a)} \qquad (4)$$

   γ = psychrometric constant = 0.66 mb/˚C
   T_s = temperature at water surface (˚C)
   T_a = temperature in atmosphere (˚C)
   e_s = vapor pressure at water surface (mb)
   e_a = vapor pressure in atmosphere (mb)

   e. now $\dfrac{Q_h}{Q_e} \equiv \dfrac{K}{E}$, hence from (4)

   $K = ER$ (5)

   f. substituting (5) into (3) we get

   $H = ER + E = E\,(R+1)$ (6)

   reorganizing (6)

   $$\frac{H}{E} = R+1 = 1 + \gamma \frac{(T_s - T_a)}{(e_s - e_a)} \qquad (7)$$

g. the basic mass-transfer equation is

$$E = f(u)\,(e_s - e_a) \qquad (8)$$

$f(u)$ = a function of windspeed, u

h. Penman proposed that we could write the mass-tranfer equation as

$$E_a = f(u)\,(e_a' - e_d) \qquad (9)$$

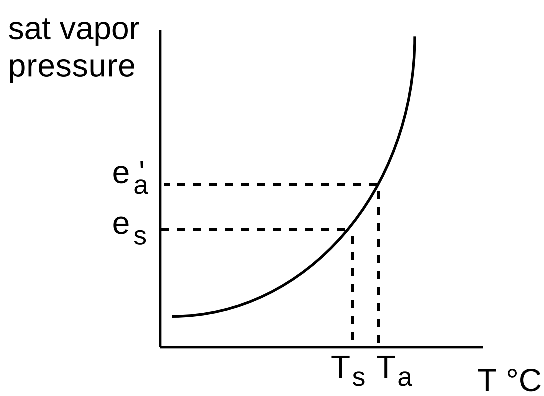
E_a = contribution of mass transfer to evaporation (cm)
e_a' = saturation vapor pressure of water surface with temperature T equal to air temperature (mb)
e_d = saturation vapor pressure of atmosphere (mb)

He *assumed* that the windspeed function $f(u)$ would be the same for both (8) and (9)

i. if we divide E_a by E we get

$$\frac{E_a}{E} = \frac{(e_a' - e_d)}{(e_s - e_a)} = 1 - \frac{e_s - e_a'}{e_s - e_d} \qquad (10)$$

j. the saturation vapor pressures e_a' and e_s increase with increasing temperature



The approximate slope at T_a of the saturation vapor pressure vs. temperature curve above is

$$\Delta = \frac{e_s - e_a'}{T_s - T_a} \qquad (11)$$

k. thus

$$T_s - T_a = \frac{e_s - e_a'}{\Delta} \qquad (12)$$

l. substituting this expression into (7) yields

$$\frac{H}{E} = 1 + \gamma \frac{\left(\frac{e_s - e_a'}{\Delta}\right)}{e_s - e_a} = 1 + \frac{\gamma\,(e_s - e_a')}{\Delta\,(e_s - e_a)} \qquad (13)$$

m. but from (10) we get

$$\frac{e_s - e_a'}{e_s - e_d} = 1 - \frac{E_a}{E} \qquad (14)$$

n. substituting (14) into (13)

$$\frac{H}{E} = 1 + \frac{\gamma}{\Delta}\left(1 - \frac{E_a}{E}\right)$$

and multiplying both sides of the equation by Δ

$$\frac{H\Delta}{E} = \Delta + \gamma - \frac{\gamma E_a}{E} \qquad (15)$$

o. solving (15) for E yields

$$E = \frac{H\Delta + \gamma E_a}{\Delta + \gamma} \qquad (16)$$

p. dividing numerator and denominator of the right side of (16) by γ yields

$$E = \frac{\frac{\Delta}{\gamma} H + E_a}{\frac{\Delta}{\gamma} + 1} \qquad (17)$$

q. Penman developed an empirical relation for E_a

$$E_a \text{ (mm/day)} = 0.47\,(0.5 + 0.01\,u_2)\,(e_a' - e_2) \qquad (18)$$

u_2 = wind velocity at 2m above the water surface (mi/day)
e_2 = vapor pressure 2m above the water surface (mb)

r. $\frac{\Delta}{\gamma}$ is a function of temperature alone and has been tabulated (e.g., Dunne & Leopold Table 4-6, and Fig. 4-8)

s. Thus we can compute E without needing the surface temperature T_s, which is hard to measure.

$H = \frac{Q_n}{\rho L}$ can be measured directly or estimated from empirical equations

t. In a *real* pan, there is significant transfer by conduction and radiation through pan walls. Kohler et. al. (1955) modified the Penman equation to account for this and produced graphs to calculate evaporation for both pans and lakes (Dunne & Leopold Fig. 4-9).

3. Penman adapted his equation to yield estimates of potential evapotranspiration by assuming:

1) for periods of a day or longer, changes of energy stored in plants and soil can be neglected
2) advected energy input is small and may be neglected

a. with these simplifications, we can write

$$Q_n = Q_h + Q_{et} \qquad (19)$$

$$\left.\begin{array}{l} Q_n = \textit{net} \text{ solar radiation} \\ Q_h = \text{energy transferred from vegetation to air as sensible heat (conduction)} \\ Q_e = \text{energy used for evapotranspiration} \end{array}\right\} \text{cal/cm}^2$$

b. as before, we apply the Bowen Ratio, R

$$R = \frac{Q_h}{Q_{et}} = \gamma \frac{(T_s - T_a)}{(e_s - e_a)} \qquad (20)$$

T_s = temperature of leaves and soil (˚C)
T_a = temperature of air (˚C)
e_s = vapor pressure of leaves and soil (mb)
e_a = vapor pressure of air (mb)

c. solving (20) for Q_h and substituting into (19)

$$Q_n = Q_{et} + RQ_{et} = Q_{et}(1 - R) \qquad (21)$$

d. solving (21) for Q_{et}

$$Q_{et} = \frac{Q_n}{1+R} = \frac{Q_n}{1 + \gamma \frac{(T_s - T_a)}{(e_s - e_a)}} \qquad (22)$$

e. we divide both sides of (22) by ρL (compare eq. 2) to get the evapotranspiration,ET

$$ET = \frac{1}{\rho L}\left(\frac{Q_n}{1 + \gamma \frac{(T_s - T_a)}{(e_s - e_a)}}\right) \qquad (23)$$

f. It is nearly impossible to determine e_s and T_s, however, because of different amounts of shade and heat flux over short distances. Instead we can try:

$$ET = \frac{1}{\rho L}\left(\frac{Q_n}{1 + \gamma \frac{(T_2 - T_1)}{(e_2 - e_1)}}\right) \qquad (24)$$

where the subscripts 1 and 2 refer to the quantities measured at two different elevations *over* the vegetated surface (e.g., at 1 and 2 m). The quantities are still difficult to measure, however.

g. If we take an area surrounded by moist, uniform terrain, then the lower layers of the atmosphere will be in approximate thermal equilibrium with the surface, and $Q_h \approx 0$. In this case, $Q_n = Q_{et}$.