

An insurance company offers four different deductible levels – none, low, medium and high – for its homeowner’s policy and three different levels – low, medium, and high – for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance. For example, the proportion of individuals with both low homeowner’s deductible and low auto deductible is .06 (6% of all individuals).

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

Suppose an individual; having both types of policies is randomly selected.

First defining events with symbols

- Let A_L = {the individual has a low auto deductible}
- Let A_M = {the individual has a medium auto deductible}
- Let A_H = {the individual has a high auto deductible}
- Let H_N = {the individual has no homeowner’s deductible}
- Let H_L = {the individual has a low homeowner’s deductible}
- Let H_M = {the individual has a medium homeowner’s deductible}
- Let H_H = {the individual has a high homeowner’s deductible}

Before beginning the problem look at the chart and see if the events are disjoint and if they all sum to 1. In this problem the events are disjoint and sum to 1.

A) What is the probability that the individual has a medium auto deductible and a high homeowner’s deductible?

Let $A_M \cap H_H = \{ \text{the individual has medium auto and high homeowner's deductible s} \}$

$$P(A_M \cap H_H) = .10$$

The chart can be used to determine the probability of the event. The chart below is highlighted to indicate the event and its probability.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

B) What is the probability that the individual has a low auto deductible?

At the beginning of the problem, the event associated with a low auto deductible was defined as (A_L). The probability of the individual having low auto deductible is 0.19. This probability was obtained by summing the disjoint events containing low auto deductibles on the chart

$$A_L = \{A_L \cap H_N\} \cup \{A_L \cap H_L\} \cup \{A_L \cap H_M\} \cup \{A_L \cap H_H\}$$

$$P(A_L) = P(A_L \cap H_N) + P(A_L \cap H_L) + P(A_L \cap H_M) + P(A_L \cap H_H)$$

(These events can be summed since these are disjoint events)

$$P(A_L) = .04 + .06 + .05 + .03 = .19$$

. Each of these events is highlighted in the chart below.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

What is the probability that the individual has a low homeowner's deductible?

At the beginning the event associated with a low homeowner's deductible was defined as (H_L).

$$H_L = \{A_L \cap H_L\} \cup \{A_M \cap H_L\} \cup \{A_L \cap H_L\}$$

$$P(H_L) = P(A_L \cap H_L) + P(A_M \cap H_L) + P(A_L \cap H_L)$$

$$P(H_L) = .06 + .10 + .03 = 0.19$$

The probability of an individual having a low homeowner's deductible is 0.19. This probability was obtained as in the first part of the question, by summing the disjoint

events associated with low homeowner's deductibles. These events are highlighted in the chart below.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

C) What is the probability that the individual is in the same category for both auto and homeowner's deductibles?

Let (Same) = {the individual is in the same category for both auto and homeowner's deductibles}

$$\text{Same} = \{A_L \cap H_L\} \cup \{A_M \cap H_M\} \cup \{A_H \cap H_H\}$$

$$P(\text{Same}) = P(A_L \cap H_L) + P(A_M \cap H_M) + P(A_H \cap H_H)$$

$$P(\text{Same}) = .06 + .20 + .15 = .41$$

The probability that the individual is in the same category is 0.41, which is obtained by summing the disjoint events from the chart. The events where the individual has the same deductible for both auto and homeowner's are highlighted in the chart below.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

D) Based on the answer in part C what is the probability that the two categories are different?

Let Different = {the individual is in different categories for auto and homeowners }

$$P(\text{Different}) = 1 - P(\text{Same})$$

$$P(\text{Different}) = 1 - 0.41 = .59$$

The probability for all events is one. To determine the probability for the deductibles being in different categories, the known probability for the deductibles being the same was subtracted from one. The events where the two categories are different are highlighted in the chart below. Notice this is the compliment of the previous problem.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

E) What is the probability that the individual has at least one low deductible?

Let Low = {the individual has at least one low deductible}

$$\text{Low} = \{A_L \cap H_N\} \cup \{A_L \cap H_L\} \cup \{A_L \cap H_M\} \cup \{A_L \cap H_H\} \cup \{A_M \cap H_L\} \cup \{A_H \cap H_L\}$$

$$P(\text{Low}) = P(A_L \cap H_N) + P(A_L \cap H_L) + P(A_L \cap H_M) + P(A_L \cap H_H) + P(A_M \cap H_L) + P(A_H \cap H_L)$$

$$P(\text{Low}) = .04 + .06 + .05 + .03 + .10 + .03 = .31$$

The probability that the individual has a least one low deductible is 0.31. The probability was obtained by summing the probability of each disjoint event containing at least one low deductible listed on the chart. These events are highlighted on the chart below.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

Another way to determine the probability is shown below.

$$P(\text{Low}) = P(A_L \cup H_L) = P(A_L) + P(H_L) - P(A_L \cap H_L)$$

$$P(\text{Low}) = .18 + .19 - .06 = .31$$

This equation sums the probability of a low auto deductible and a low homeowner's deductible, which were already calculated in part B, then subtracts the probability of both auto and homeowner's of being low to avoid counting it twice. The probability of both auto and homeowner's being low is highlighted on the chart below.

Auto	Homeowner's			
	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

F) Using the answer in part C, what is the probability that neither deductible level is low?

Let Neither = {neither deductible is low}

$$P(\text{Neither}) = 1 - P(\text{Low})$$

$$P(\text{Neither}) = 1 - .39 = .69$$

Since all events add up to one, to find the compliment of both deductibles being low, the known probability for both being low is subtracted from one.