2026 MCM Problem A: Modeling Smartphone Battery Drain

Chase Loughmiller, Devon Walker, and Jonathan David Juarez
Department of Mathematics & Data Science
California State Polytechnic University, Humboldt

Introduction

Smartphones have become indispensable in modern life, yet users frequently experience inconsistent battery performance. Under seemingly similar usage patterns, a device may last an entire day or drain within a few hours. Understanding this behavior requires modeling how energy is consumed under varying usage conditions.

Lithium-ion batteries power nearly all smartphones, and their performance depends on several interacting factors including:

- device usage patterns
- battery health
- electrical current demand
- device temperature
- capacity degradation over time

The goal of this research is to construct a continuous-time mathematical model describing the state of charge (SOC) of a smartphone battery and to analyze how different user behaviors affect battery longevity and time-to-empty (TTE).

Objective

The primary objectives of this study are:

1. Develop a mathematical model describing smartphone battery discharge over time.
2. Represent the state of charge (SOC) as a continuous-time function.
3. Incorporate state of health (SOH) and temperature effects into battery degradation.
4. Evaluate how different usage scenarios (calls, gaming, video, web browsing) influence battery life.
5. Estimate the time-to-empty (TTE) under realistic usage conditions.

Conceptual Framework

The model is built on the principle that battery charge evolves according to the electrical current drawn by the device. The state of charge (SOC) is defined as:

$$SOC = \frac{Q(t)\ \text{charge stored in the battery at time } t}{Q_{nom}\ \text{nominal battery capacity}}$$

Because electrical current represents the rate of charge flow:

$$Q(t) = Q(t_0) + \int_{t_0}^{t} I(t)dt$$

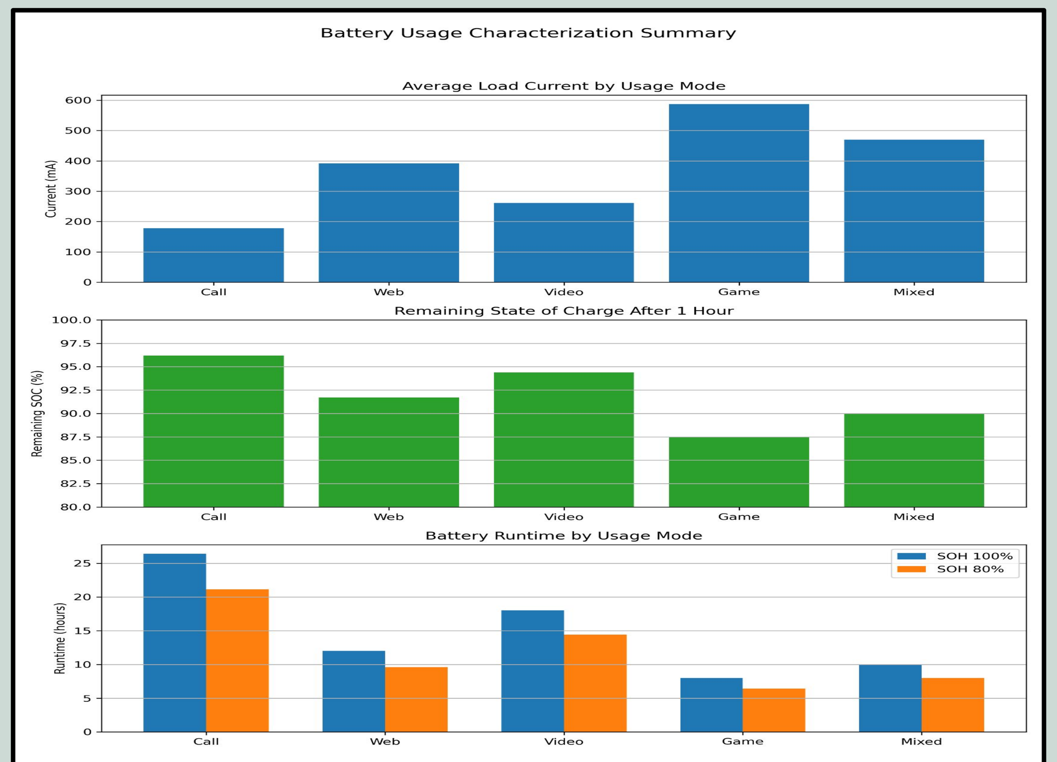
Substituting into the SOC definition gives the continuous-time SOC equation:

$$SOC(t) = SOC(t_0) + \int_{t_0}^{t} \frac{I(t)}{Q_{nom}} dt$$

This formulation allows battery discharge to be modeled under varying power demands.

Model Framework, Findings, and Model Validation

A continuous-time battery discharge model was developed using activity-based current demand and temperature-dependent aging. Simulations compare healthy (SOH = 1.0) and degraded (SOH ≈ 0.8) batteries across common smartphone workloads. Results show that high-energy activities drive the fastest depletion, while reduced battery health and elevated temperatures significantly shorten runtime. Model predictions align with benchmark data and reproduce realistic discharge behavior, providing a practical framework for estimating battery life under varying conditions.



Funcationality	Est. TTE (hrs.)	Conf. TTE (hrs.)	Diff.	Avg. Current (mA)	dSOC/dt
Call	26.43	26.43	0	177.8	-1.05E-05
Video	15.35	15.35	0	306.2	-2.32E-05
Web	11.58	11.58	0	405.8	-1.54E-05
Game	8.55	8.55	0	549.7	-3.47E-05
Mixed	13.07	13.07	0	359.7	-2.78E-05

Figure 5: Battery usage characterization summary.

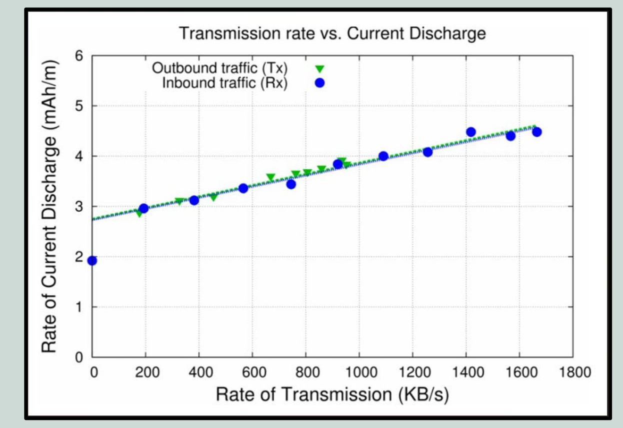


Figure 1: Current Draw as a Function of Data Transmission Rate for Inbound and Outbound Traffic

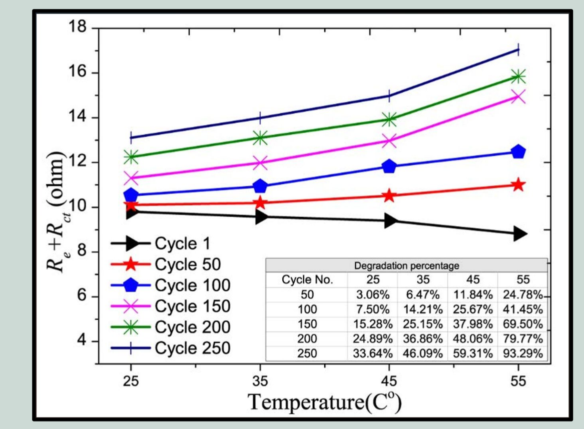


Figure 2: Temperature-Dependent Growth of Internal Battery Resistance Across Charge Cycles

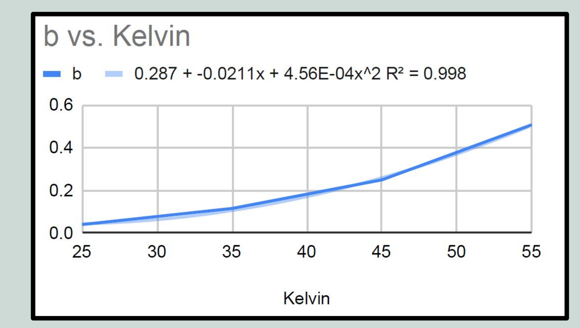


Figure 3: Temperature Dependence of the Resistance-Growth Slope Parameter m(T)

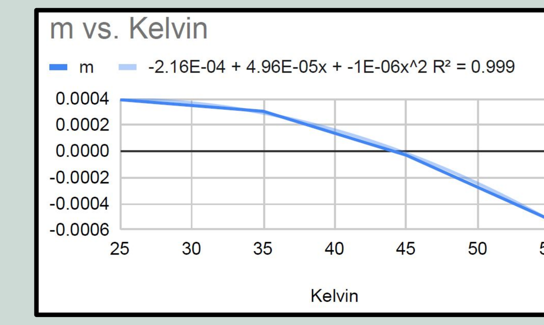


Figure 4: Temperature Dependence of the Resistance-Growth Intercept Parameter b(T)

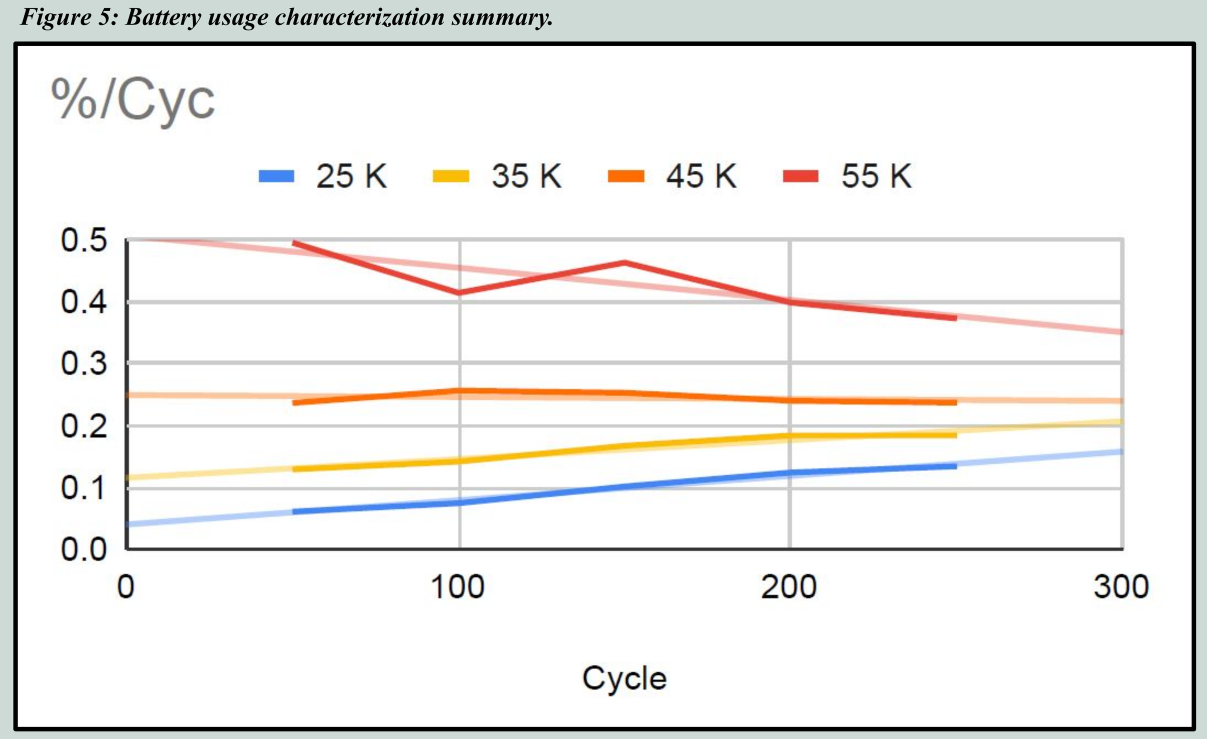


Figure 6: Cycle-by-Cycle Percent Increase in Internal Resistance at Four Operating Temperatures

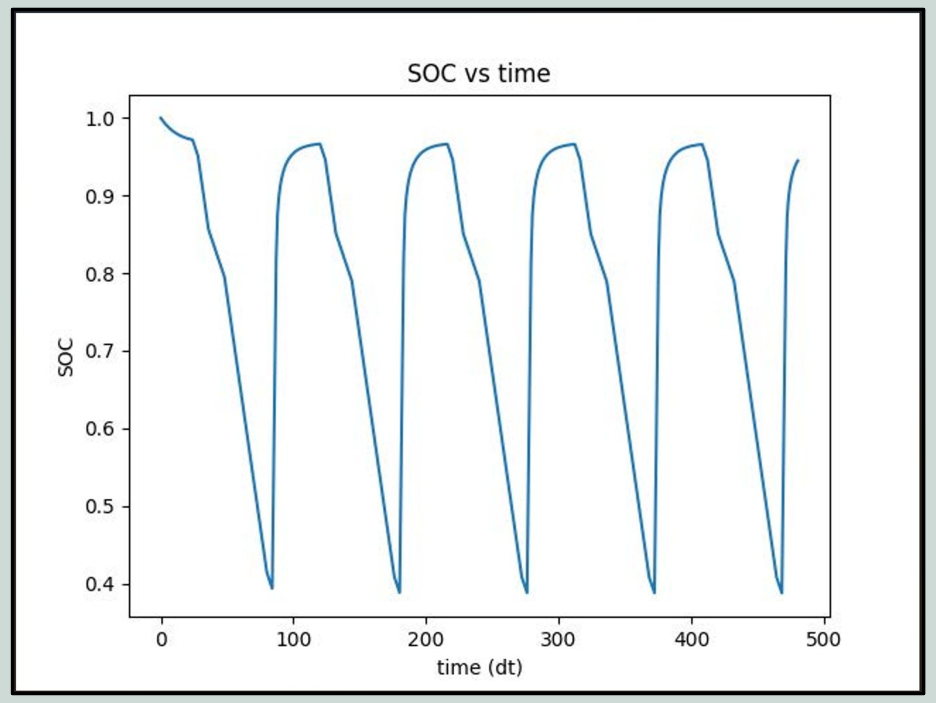


Figure 7: SOC Behavior Under Low-Wattage Charging: Inability to Reach Full Charge Due to Standby Drain

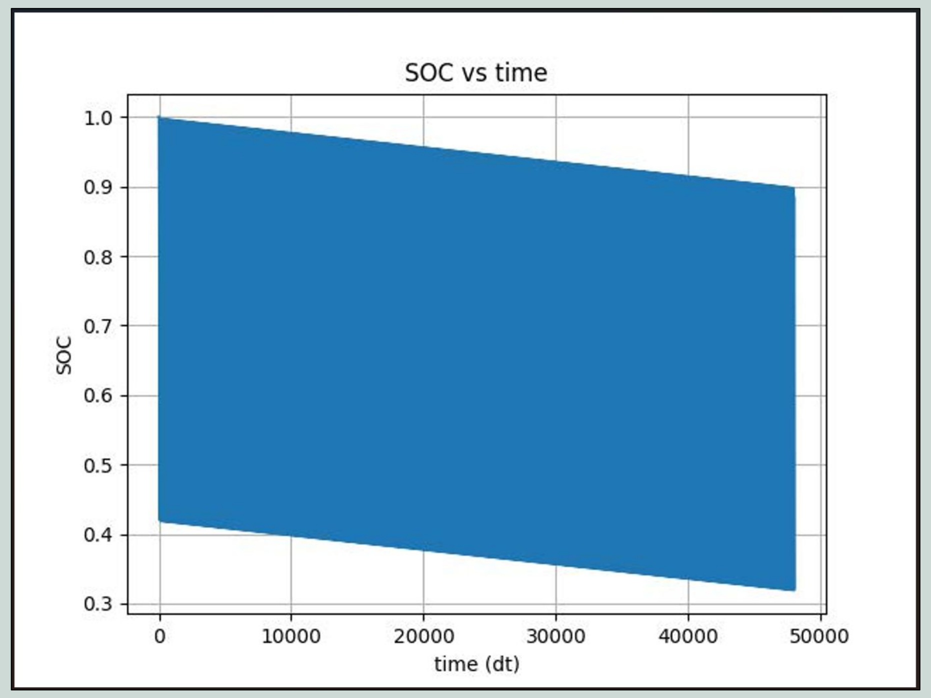


Figure 8: Long-Term Capacity Loss Demonstrated Through SOC Decline Over Hundreds of Charge Cycles

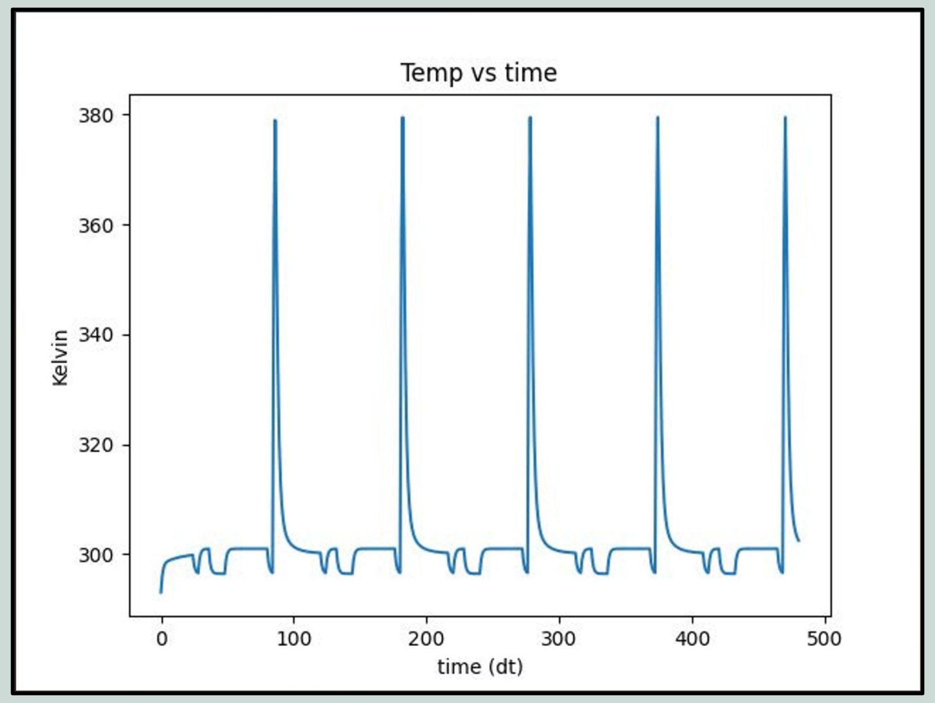


Figure 9: Device Temperature Dynamics Showing Heat Spikes During High-Power Charging.

Results

The results demonstrate that:

- User activity patterns strongly influence battery longevity.
- Battery health degradation significantly impacts performance.
- Temperature-driven aging accelerates long-term capacity loss.

Such models can support improved energy management strategies, smarter operating systems, and user recommendations for maximizing battery life.

Model Limitations and Future Work

While the framework captures the primary drivers of battery depletion, it assumes fixed usage states, constant hardware behavior, and uniform temperature conditions within the device. In practice, smartphone power consumption varies continuously due to background processes, dynamic processor workloads, and adaptive display settings.

Future work will extend the model by introducing time-varying current demand and additional variables representing processor activity, wireless communication, and environmental temperature. These extensions will allow simulations of realistic mixed-usage patterns and long-term degradation across charge cycles, producing a more predictive framework for estimating smartphone battery performance.

$$\frac{dSOC}{dt} = -\frac{I(T)}{Q_{nom}} - f(T,t) - g(N_{cycles})$$

Acknowledgements

We would like to thank the Mathematical Contest in Modeling (MCM) for the opportunity to participate in this competition. We are especially grateful to Dr. Larripa for the guidance, encouragement, and support that motivated us to keep pushing forward throughout this project.

References

Contact Card